

# Strong couplings of negative and positive parity nucleons to the heavy baryons and mesons

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## Abstract

The strong coupling form factors related to the strong vertices of the positive and negative parity nucleons with the heavy  $\Lambda_{b[c]}$  baryons and heavy  $B^*[D^*]$  vector mesons are calculated using a three-point correlation function. Using the values of the form factors at  $Q^2 = -m_{meson}^2$  we compute the strong coupling constants among the participating particles.

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# 1 Introduction

The recently achieved progresses in experimental sector related to the charm and bottom baryons have provided important clues and motivations for the theoretical studies on this area. The necessity for a better understanding of the properties of these baryons such as their masses, structures and interactions with other particles have increased the theoretical interests on them. Their various properties were studied using different methods. For instance their masses were studied in Refs. [1–5] (see also the references therein) via various methods such as quenched lattice non-relativistic QCD, the QCD sum rule approach within the framework of heavy quark effective theory, the constituent quark model, QCD sum rules and a theoretical approach based on modeling the color hyperfine interaction. The Refs. [6–19] and references therein provide some examples in which their strong and weak decays were studied.

This work provides an analysis of the strong couplings of the heavy  $\Lambda_{b(c)}$  and  $\Sigma_{b(c)}$  baryons to the positive parity nucleon  $N$ / negative parity nucleon  $N^*$  and heavy  $B^*$  /  $D^*$  vector meson. Here by  $N^*$  we mean the excited  $N(1535)$  nucleon with  $J^P = \frac{1}{2}^-$ . Such couplings occur in a low energy regime that preclude us from the usage of the perturbative approach. The strong coupling constants are the basic parameters to determine the strength of the strong interactions among the participated particles. They also provide us a better understanding on the structure and nature of the hadrons participated in the interaction. To improve our understanding on the perturbative and non-perturbative natures of the strong interaction they can also provide valuable insights. Furthermore, these coupling constants may be useful for explanation of the observation of various exotic events by different collaborations. Beside these, one may resort to these results in order to explain the properties of  $B^*$  and  $D^*$  mesons in nuclear medium. The nucleon cloud may affect properties of these mesons such as their masses and decay constants in nuclear medium due to their interactions with nucleons (see for instance the Refs. [20–25]). Therefore, the present study is also helpful to identify the properties of these particles in nuclear medium.

Here, we calculate the strong form factors defining the strong vertices  $\Lambda_b N B^*$ ,  $\Lambda_b N^* B^*$ ,  $\Sigma_b N B^*$ ,  $\Sigma_b N^* B^*$ ,  $\Lambda_c N D^*$ ,  $\Lambda_c N^* D^*$ ,  $\Sigma_c N D^*$  and  $\Sigma_c N^* D^*$  in the framework of the QCD sum rule [26] as one of the powerful and applicable non-perturbative tools to hadron physics. By using  $Q^2 = -m_{meson}^2$ , we then obtain the strong coupling constants among the participating particles. This method has been previously applied to investigate some other vertices (for instance see Refs. [6, 17, 27–29] and references therein).

The paper contains three sections. In next section, we calculate the strong coupling form factors in the context of QCD sum rule approach. Section 3 is devoted to the numerical analysis of the results and discussion.

## 2 The strong coupling form factors

In this section we calculate the coupling form factors defining the vertices among the hadrons under consideration using the QCD sum rule method. The starting point is to

consider the following three-point correlation function:

$$\Pi_\mu(q) = i^2 \int d^4x \int d^4y e^{-ip \cdot x} e^{ip' \cdot y} \langle 0 | \mathcal{T} (J_N(y) J_{\mathcal{M}}^\mu(0) \bar{J}_{\mathcal{B}}(x)) | 0 \rangle, \quad (1)$$

where  $\mathcal{T}$  is the time ordering operator and  $q = p - p'$  is the transferred momentum. In this equation  $J_i$  denote the interpolating fields of different particles,  $\mathcal{M}$  symbolizes the  $B^*$  or  $D^*$  meson,  $\mathcal{B}$  stands for the  $\Lambda_{b(c)}$  or  $\Sigma_{b(c)}$  baryons and  $N$  shows the nucleon with both parities.

The three-point correlation function can be calculated both in terms of the hadronic degrees of freedom and in terms of the QCD degrees of freedom. These two different ways of calculations are called as physical and OPE sides, respectively. The results obtained from both sides are equated to acquire the QCD sum rules for the coupling form factors. For the suppression of the contributions coming from the higher states and continuum a double Borel transformation with respect to the variables  $p^2$  and  $p'^2$  are applied to both sides of the obtained sum rules.

## 2.1 Physical Side

For the physical side of the calculation one inserts complete sets of appropriate  $\mathcal{M}$ ,  $\mathcal{B}$  and  $N$  hadronic states, which have the same quantum numbers as the corresponding interpolating currents, into the correlation function. Integrals over  $x$  and  $y$  give

$$\begin{aligned} \Pi_\mu^{Phy}(q) &= \frac{\langle 0 | J_N | N(p', s') \rangle \langle 0 | J_{\mathcal{M}}^\mu | \mathcal{M}(q) \rangle \langle N(p', s') \mathcal{M}(q) | \mathcal{B}(p, s) \rangle \langle \mathcal{B}(p, s) | \bar{J}_{\mathcal{B}} | 0 \rangle}{(p^2 - m_{\mathcal{B}}^2)(p'^2 - m_N^2)(q^2 - m_{\mathcal{M}}^2)} \\ &+ \frac{\langle 0 | J_N | N^*(p', s') \rangle \langle 0 | J_{\mathcal{M}}^\mu | \mathcal{M}(q) \rangle \langle N^*(p', s') \mathcal{M}(q) | \mathcal{B}(p, s) \rangle \langle \mathcal{B}(p, s) | \bar{J}_{\mathcal{B}} | 0 \rangle}{(p^2 - m_{\mathcal{B}}^2)(p'^2 - m_{N^*}^2)(q^2 - m_{\mathcal{M}}^2)} \\ &+ \dots, \end{aligned} \quad (2)$$

where  $\dots$  stands for the contributions coming from the higher states and continuum and the contributions of both positive and negative parity nucleons have been included. The matrix elements in this equation are parameterized as

$$\begin{aligned} \langle 0 | J_N | N(p', s') \rangle &= \lambda_N u_N(p', s'), \\ \langle 0 | J_N | N^*(p', s') \rangle &= \lambda_{N^*} \gamma_5 u_{N^*}(p', s'), \\ \langle \mathcal{B}_{b(c)}(p, s) | \bar{J}_{\mathcal{B}_{b(c)}} | 0 \rangle &= \lambda_{\mathcal{B}_{b(c)}} \bar{u}_{\mathcal{B}_{b(c)}}(p, s), \\ \langle 0 | J_{\mathcal{M}}^\mu | \mathcal{M}(q) \rangle &= m_{\mathcal{M}} f_{\mathcal{M}} \epsilon_\mu^*, \\ \langle N(p', s') \mathcal{M}(q) | \mathcal{B}(p, s) \rangle &= \epsilon^\nu \bar{u}_N(p', s') \left[ g_1 \gamma_\nu - \frac{i \sigma_{\nu\alpha} q^\alpha}{m_{\mathcal{B}} + m_N} g_2 \right] u_{\mathcal{B}}(p, s), \\ \langle N^*(p', s') \mathcal{M}(q) | \mathcal{B}(p, s) \rangle &= \epsilon^\nu \bar{u}_{N^*}(p', s') \gamma_5 \left[ g_1^* \gamma_\nu - \frac{i \sigma_{\nu\alpha} q^\alpha}{m_{\mathcal{B}} + m_{N^*}} g_2^* \right] u_{\mathcal{B}}(p, s), \end{aligned} \quad (3)$$

where  $\lambda_{N(N^*)}$  and  $\lambda_{\mathcal{B}}$  are the residues of the related baryons,  $u_{N(N^*)}$  and  $u_{\mathcal{B}}$  are the spinors for the nucleon,  $\Lambda_b(\Lambda_c)$  and  $\Sigma_b(\Sigma_c)$  baryons; and  $f_{\mathcal{M}}$  represents the leptonic decay constant of  $B^*(D^*)$ . Here  $g_1$  and  $g_2$  are strong coupling form factors related to the couplings of

the  $\mathcal{B}$  baryon and  $\mathcal{M}$  meson to the positive parity nucleon  $N$ ; and  $g_1^*$  and  $g_2^*$  are those related to the strong vertices of  $\mathcal{B}$  baryon and  $\mathcal{M}$  meson with the negative parity nucleon  $N^*$ . Application of the double Borel transformation with respect to the initial and final momenta squared yields

$$\widehat{\mathbf{B}}\Pi_\mu^{Phy}(q) = \lambda_{\mathcal{B}} f_{\mathcal{M}} e^{-\frac{m_{\mathcal{B}}^2}{M^2}} e^{-\frac{m_N^2 + m_{N^*}^2}{M'^2}} [\Phi_1 \gamma_\mu + \Phi_2 \not{p} q_\mu + \Phi_3 \not{q} p_\mu + \Phi_4 \not{q} \gamma_\mu] + \dots, \quad (4)$$

where

$$\begin{aligned} \Phi_1 &= \frac{m_{\mathcal{M}}}{(m_{\mathcal{B}} + m_{N^*})(m_{\mathcal{M}}^2 - q^2)} \left[ e^{\frac{m_N^{*2}}{M'^2}} \lambda_N (g_1 + g_2) (m_{\mathcal{B}} + m_{N^*}) (-m_N^2 + m_N m_{\mathcal{B}} + q^2) \right. \\ &\quad \left. + e^{\frac{m_N^2}{M'^2}} \lambda_{N^*} (g_1^* (m_{\mathcal{B}} + m_{N^*}) + g_2^* (m_{\mathcal{B}} - m_{N^*})) (m_{N^*}^2 + m_{N^*} m_{\mathcal{B}} - q^2) \right], \\ \Phi_2 &= \frac{1}{m_{\mathcal{M}} (m_{\mathcal{B}} + m_{N^*})(m_{\mathcal{M}}^2 - q^2)} \left[ e^{\frac{m_N^{*2}}{M'^2}} \lambda_N (g_1 (m_N^2 - m_{\mathcal{B}}^2) + g_2 m_{\mathcal{M}}^2) (m_{\mathcal{B}} + m_{N^*}) \right. \\ &\quad \left. + e^{\frac{m_N^2}{M'^2}} \lambda_{N^*} (m_{\mathcal{B}} - m_{N^*}) (g_1^* (m_{\mathcal{B}} + m_{N^*})^2 - g_2^* m_{\mathcal{M}}^2) \right], \\ \Phi_3 &= -\frac{2m_{\mathcal{M}}}{(m_{\mathcal{M}} + m_N)(m_{\mathcal{B}} + m_{N^*})(m_{\mathcal{M}}^2 - q^2)} \left[ e^{\frac{m_N^{*2}}{M'^2}} \lambda_N (m_{\mathcal{B}} + m_{N^*}) (g_1 (m_{\mathcal{B}} + m_N) + g_2 m_N) \right. \\ &\quad \left. - e^{\frac{m_N^2}{M'^2}} \lambda_{N^*} (m_{\mathcal{B}} + m_N) (g_1^* (m_{\mathcal{B}} + m_{N^*}) - g_2^* m_{N^*}) \right], \\ \Phi_4 &= \frac{m_{\mathcal{B}} m_{\mathcal{M}}}{(m_{\mathcal{B}} + m_{N^*})(m_{\mathcal{M}}^2 - q^2)} \left[ -e^{\frac{m_N^{*2}}{M'^2}} \lambda_N (g_1 + g_2) (m_{\mathcal{B}} + m_{N^*}) \right. \\ &\quad \left. + e^{\frac{m_N^2}{M'^2}} \lambda_{N^*} (g_1^* (m_{\mathcal{B}} + m_{N^*}) + g_2^* (m_{\mathcal{B}} - m_{N^*})) \right], \end{aligned} \quad (5)$$

with  $M^2$  and  $M'^2$  being the Borel mass parameters.

## 2.2 OPE Side

For the OPE side of the calculation, the basic ingredients are the explicit expressions of the interpolating currents in terms of the quark fields, which are taken as

$$\begin{aligned} J_{\Lambda_{b[c]}}(x) &= \epsilon_{abc} u^{aT}(x) C \gamma_5 d^b(x) (b[c])^c(x), \\ J_{\Sigma_{b[c]}}(x) &= \epsilon_{abc} \left( u^{aT}(x) C \gamma_\nu d^b(x) \right) \gamma_5 \gamma_\nu (b[c])^c(x), \\ J_N(y) &= \epsilon_{ij\ell} \left( u^{iT}(y) C \gamma_\beta u^j(y) \right) \gamma_5 \gamma_\beta d^\ell(y), \\ J_{B^*[D^*]}(0) &= \bar{u}(0) \gamma_\mu b[c](0), \end{aligned} \quad (6)$$

with  $C$  being the charge conjugation operator. By replacing these interpolating currents in Eq. (1) and doing contractions of all quark pairs via Wick's theorem, we get

$$\Pi_\mu^{OPE}(q) = i^2 \int d^4x \int d^4y e^{-ip \cdot x} e^{ip' \cdot y} \epsilon_{abc} \epsilon_{ij\ell}$$

$$\begin{aligned}
& \times \left\{ \gamma_5 \gamma_\beta S_d^{cj}(y-x) \gamma_5 C S_u^{bi^T}(y-x) C \gamma_\beta S_u^{ah}(y) \gamma_\mu S_{b[c]}^{h\ell}(-x) \right. \\
& \left. - \gamma_5 \gamma_\beta S_d^{cj}(y-x) \gamma_5 C S_u^{ai^T}(y-x) C \gamma_\beta S_u^{bh}(y) \gamma_\mu S_{b[c]}^{h\ell}(-x) \right\}, \quad (7)
\end{aligned}$$

for  $\Lambda_b N^{(*)} B^*$  and  $\Lambda_c N^{(*)} D^*$  vertices and

$$\begin{aligned}
\Pi_\mu^{OPE}(q) &= i^2 \int d^4x \int d^4y e^{-ip \cdot x} e^{ip' \cdot y} \epsilon_{abc} \epsilon_{ij\ell} \\
&\times \left\{ \gamma_5 \gamma_\beta S_d^{cj}(y-x) \gamma_\nu C S_u^{bi^T}(y-x) C \gamma_\beta S_u^{ah}(y) \gamma_\mu S_{b[c]}^{h\ell}(-x) \gamma_\nu \gamma_5 \right. \\
&\left. - \gamma_5 \gamma_\beta S_d^{cj}(y-x) \gamma_\nu C S_u^{ai^T}(y-x) C \gamma_\beta S_u^{bh}(y) \gamma_\mu S_{b[c]}^{h\ell}(-x) \gamma_\nu \gamma_5 \right\}, \quad (8)
\end{aligned}$$

for  $\Sigma_b N^{(*)} B^*$  and  $\Sigma_c N^{(*)} D^*$  vertices. In these equations,  $S_{b[c]}^{ij}(x)$  and  $S_{u[d]}^{ij}(x)$  correspond to the heavy and light quark propagators, respectively. Using the heavy and light quark propagators in coordinate space and after lengthy calculations (for details see Refs. [28, 29]), we obtain

$$\begin{aligned}
\Pi_\mu^{OPE}(q) &= \Pi_1^{OPE}(q^2) \gamma_\mu + \Pi_2^{OPE}(q^2) \not{p} q_\mu + \Pi_3^{OPE}(q^2) \not{q} p_\mu + \Pi_4^{OPE}(q^2) \not{q} \gamma_\mu \\
&+ \text{other structures}, \quad (9)
\end{aligned}$$

where the  $\Pi_i(q^2)$  functions contain contributions coming from both the perturbative and non-perturbative parts and are given as

$$\Pi_i^{OPE}(q^2) = \int ds \int ds' \frac{\rho_i^{pert}(s, s', q^2) + \rho_i^{non-pert}(s, s', q^2)}{(s - p^2)(s' - p'^2)}. \quad (10)$$

The spectral densities  $\rho_i(s, s', q^2)$  appearing in this equation are obtained from the imaginary parts of the  $\Pi_i$  functions as  $\rho_i(s, s', q^2) = \frac{1}{\pi} \text{Im}[\Pi_i]$ . Here, as examples, only the results of the spectral densities corresponding to the Dirac structure  $\gamma_\mu$  for  $\Lambda_b N B^*$  vertex are presented, which are

$$\begin{aligned}
\rho_1^{pert}(s, s', q^2) &= \frac{m_b m_u s'^2}{64\pi^4 \mathcal{Q}} \Theta \left[ L_1(s, s', q^2) \right] + \int_0^1 dx \int_0^{1-x} dy \frac{1}{64\pi^4 \mathcal{X}^3} \left\{ m_b^4 x^2 (\mathcal{X}' + 2x) \right. \\
&\times (\mathcal{X} + y) + q^4 xy \left[ 3y(\mathcal{X}' - 1) \mathcal{X}'(\mathcal{X}' + 4x) - 2\mathcal{X}'^2(3x + \mathcal{X}') + 2y^2(15x^2 - 14x + 2) \right] \\
&- 2q^2 \mathcal{X} \left[ xy \mathcal{X}' \left( s(2 + 15x^2 - 18x) - s'(\mathcal{X}' - 3) \right) + y^2 \left( 2sx(1 - 10x + 15x^2) \right. \right. \\
&- 4sx^2 \mathcal{X}'^2 - s'(1 - 21x + 41x^2 - 15x^3) \left. \left. + s'y^3(1 - 17x + 30x^2) - 4sx^2 \mathcal{X}'^2 \right] \right. \\
&+ m_b^3 x \left( m_u(3 - 5x + 2x^2 - 2xy) + 3m_d(\mathcal{X}' + x) \mathcal{X} \right) + \mathcal{X}^2 \left[ s'^2 y(5y - 8x \mathcal{X}' \right. \\
&- 34xy - 6y^2 + 15x^2 y + 30xy^2) + 3s^2 x^2(1 - 4y - 6x + 5x^2 + 10xy) + 2ss'x \\
&\times (5y - 9y^2 - 4x \mathcal{X}' + 15x^2 y - 26xy + 30xy^2) \left. \right] + 2m_b^2 x \left[ q^2 \left( 3x \mathcal{X}' - 3y + 16xy \right. \right. \\
&\left. \left. + 8x^2 y - 4y^2 + 16xy^2 \right) + \mathcal{X} \left( s'(3y - 3x \mathcal{X}' - 16y + 8xy - 4y^2 + 16xy^2) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + 2sx(1 - 3y - 5x + 4x^2 + 8xy)) \Big] + m_b \Big[ 3m_d \mathcal{X} \Big( sx\mathcal{X}(\mathcal{X}' - 1) + y(s'u - q^2x) \\
& \times (3y - 1) \Big) + m_u \Big( sx\mathcal{X}(6 - 9x + 3x^2 - 3xy) + y \Big( q^2x(4x - 3x^2 + 3xy + y - 1) \\
& + 3s'\mathcal{X}\mathcal{X}'^2 - 3s'y\mathcal{X}(\mathcal{X}' + 2) \Big) \Big) \Big] \Big\} \Theta \Big[ L_2(s, s', q^2) \Big], \tag{11}
\end{aligned}$$

and

$$\begin{aligned}
\rho_1^{non-pert}(s, s', q^2) &= \frac{\langle u\bar{u} \rangle}{16\pi^2 \mathcal{Q}} \Big[ s'(m_u - 2m_b) - q^2 m_d \Big] \Theta \Big[ L_1(s, s', q^2) \Big] \\
&+ \int_0^1 dx \int_0^{1-x} dy \frac{1}{8\pi^2 \mathcal{X}} \Big[ \langle d\bar{d} \rangle \Big( m_d(2x + \mathcal{X}')\mathcal{X} - m_b(x + \mathcal{X}\mathcal{X}') - m_u\mathcal{X} \Big) \\
&+ \langle u\bar{u} \rangle \Big( m_u(3xy + 3x\mathcal{X}' - y) - m_b(x + \mathcal{X}') - 2m_d\mathcal{X} \Big) \Big] \Theta \Big[ L_2(s, s', q^2) \Big] \\
&- \langle \alpha_s \frac{G^2}{\pi} \rangle \frac{1}{1152\pi^2 \mathcal{Q}^4} \Big[ 9m_b \mathcal{Q}^3(m_d - 2m_u) + s' \mathcal{Q}^2 \Big( 3m_b(m_d + m_u) + 2q^2 \Big) \\
&+ 3s'^2 \Big( m_b^4 - 2q^2 m_b(m_b - m_u) + q^4 \Big) \Big] \Theta \Big[ L_1(s, s', q^2) \Big] \\
&+ \int_0^1 dx \int_0^{1-x} dy \langle \alpha_s \frac{G^2}{\pi} \rangle \frac{1}{192\pi^2 \mathcal{X}^3} \Big[ 3\mathcal{X}'^3(2x + \mathcal{X}') + y^2 \Big( 15 + x(39\mathcal{X}' - 20) \Big) \\
&+ y(2x + \mathcal{X}') + \mathcal{X}'(11\mathcal{X}' - 1) + 6y^3(2x + \mathcal{X}') \Big] \Theta \Big[ L_2(s, s', q^2) \Big] \\
&- \frac{1}{192\pi^2 \mathcal{Q}} \Big[ m_0^2 \langle d\bar{d} \rangle (6m_b + 4m_d) + m_0^2 \langle u\bar{u} \rangle (7m_u - 3m_d - 18m_b) \Big] \\
&\times \Theta \Big[ L_1(s, s', q^2) \Big], \tag{12}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{X} &= x + y - 1, \\
\mathcal{X}' &= x - 1, \\
\mathcal{Q} &= m_b^2 - q^2, \\
L_1(s, s', q^2) &= s', \\
L_2(s, s', q^2) &= -m_b^2 x + sx - sx^2 + s'y + q^2 xy - sxy - s'xy - s'y^2. \tag{13}
\end{aligned}$$

The  $\Theta[...]$  in these equations is the unit-step function. As already stated, the match of the results obtained from physical and OPE sides of the correlation function gives the QCD sum rules for the strong coupling form factors. As examples, for the form factors related to the  $\Lambda_b N B^*$  and  $\Lambda_b N^* B^*$  vertices, we get

$$\begin{aligned}
g_1(q^2) &= e^{\frac{m_{\Lambda_b}^2}{M^2}} e^{\frac{m_N^2}{M'^2}} \frac{(m_{B^*}^2 - q^2)}{\lambda_N \mathcal{H}} \Big\{ m_{B^*}^2 \Big[ m_{\Lambda_b}^4 (\Pi_3 - 2\Pi_2) - 2\mathcal{V} m_N m_{N^*} \Pi_4 - 2m_{\Lambda_b}^3 (m_{N^*} \Pi_2 + \Pi_4) \\
&+ m_{\Lambda_b} \Big( 2q^2 \Pi_4 + (m_N^2 - m_N m_{N^*}) (m_{N^*} \Pi_3 + 2\Pi_4) \Big) \\
&- m_{\Lambda_b}^2 \Big( m_N m_{N^*} (2\Pi_2 - \Pi_3) + m_{N^*}^2 \Pi_3 + m_N^2 (\Pi_3 - 2\Pi_2) + 2m_{N^*} \Pi_4 - 2\Pi_1 \Big) \Big] \Big\}
\end{aligned}$$

$$\begin{aligned}
& + (m_{\Lambda_b} - m_{N^*})(m_{\Lambda_b} + m_{N^*}) \left[ m_{\Lambda_b} m_N m_{N^*} \mathcal{V} \Pi_3 - 2m_N m_{N^*} \mathcal{V} \Pi_4 + m_{\Lambda_b}^4 \Pi_3 + 2q^2 m_{\Lambda_b} \Pi_4 \right. \\
& + \left. m_{\Lambda_b}^2 \left( 2\Pi_1 - (m_N \mathcal{V} + m_{N^*}^2) \Pi_3 \right) \right] \Big\}, \\
g_2(q^2) &= e^{\frac{m_{\Lambda_b}^2}{M^2}} e^{\frac{m_{N^*}^2}{M'^2}} \frac{(m_{B^*}^2 - q^2)(m_N + m_{\Lambda_b})}{\lambda_N \mathcal{H}} \left\{ -m_{\Lambda_b}^5 \Pi_3 + m_N m_{\Lambda_b}^4 \Pi_3 + m_{\Lambda_b} m_{N^*}^3 \mathcal{V} \Pi_3 \right. \\
& + m_{N^*} m_{\Lambda_b}^3 (2m_{N^*} \Pi_3 - m_N \Pi_3 - 2\Pi_4) + 2m_{\Lambda_b} \left( m_N (m_{N^*}^2 - q^2) + m_{N^*} q^2 \right) \Pi_4 - 2m_{N^*}^3 \mathcal{V} \Pi_4 \\
& + m_{B^*}^2 (m_{\Lambda_b} - \mathcal{V}) \left( m_{\Lambda_b} (2m_{\Lambda_b} \Pi_2 - m_{\Lambda_b} \Pi_3 + m_{N^*} \Pi_3) + 2(m_{\Lambda_b} - m_{N^*}) \Pi_4 \right) \\
& - \left. m_{\Lambda_b}^2 \left( m_{N^*} (m_N m_{N^*} \Pi_3 - 4m_N \Pi_4 + 4m_{N^*} \Pi_4) + 2\mathcal{V} \Pi_1 \right) \right\}, \\
g_1^*(q^2) &= e^{\frac{m_{\Lambda_b}^2}{M^2}} e^{\frac{m_{N^*}^2}{M'^2}} \frac{(m_{B^*}^2 - q^2)}{\lambda_{N^*} \mathcal{H}} \left\{ (m_{\Lambda_b} - m_N)(m_{\Lambda_b} + m_N) \left[ m_{\Lambda_b}^4 \Pi_3 + m_{\Lambda_b} m_N m_{N^*} \mathcal{V} \Pi_3 \right. \right. \\
& + 2m_N m_{N^*} \mathcal{V} \Pi_4 + 2m_{\Lambda_b} q^2 \Pi_4 + m_{\Lambda_b}^2 \left( 2\Pi_1 - (m_N \mathcal{V} + m_{N^*}^2) \Pi_3 \right) \Big] \\
& + m_{B^*}^2 \left[ m_N m_{N^*} m_{\Lambda_b} \mathcal{V} \Pi_3 + 2m_{\Lambda_b}^3 (m_N \Pi_2 - \Pi_4) - 2m_N m_{N^*} \mathcal{V} \Pi_4 \right. \\
& + m_{\Lambda_b}^4 (\Pi_3 - 2\Pi_2) - 2m_{\Lambda_b} \left( m_{N^*} \mathcal{V} - q^2 \right) \Pi_4 + m_{\Lambda_b}^2 \left( m_{N^*}^2 (2\Pi_2 - \Pi_3) - m_N^2 \Pi_3 \right. \\
& + \left. \left. m_N (2\Pi_4 + m_{N^*} \Pi_3 - 2m_{N^*} \Pi_2) + 2\Pi_1 \right) \right] \Big\}, \\
g_2^*(q^2) &= e^{\frac{m_{\Lambda_b}^2}{M^2}} e^{\frac{m_{N^*}^2}{M'^2}} \frac{(m_{B^*}^2 - q^2)(m_{\Lambda_b} + m_{N^*})}{\lambda_{N^*} \mathcal{H}} \left\{ \left( m_{\Lambda_b}^5 + m_{\Lambda_b} m_N^3 \mathcal{V} + m_{\Lambda_b}^4 m_{N^*} \right) \Pi_3 + m_{\Lambda_b}^3 m_N \right. \\
& \times \left( (\mathcal{V} - m_N) \Pi_3 - 2\Pi_4 \right) - 2m_N^3 \mathcal{V} \Pi_4 + 2m_{\Lambda_b} \left( m_N^2 m_{N^*} + q^2 \mathcal{V} \right) \Pi_4 - m_{B^*}^2 (m_{\Lambda_b} - \mathcal{V}) \\
& \times \left[ m_{\Lambda_b} \left( 2m_{\Lambda_b} \Pi_2 - (m_{\Lambda_b} + m_N) \Pi_3 \right) + 2(m_{\Lambda_b} + m_N) \Pi_4 \right] - m_{\Lambda_b}^2 \left[ m_N \left( m_N m_{N^*} \Pi_3 \right. \right. \\
& - \left. \left. 4m_N \Pi_4 + 4m_{N^*} \Pi_4 \right) - 2\mathcal{V} \Pi_1 \right] \Big\}, \tag{14}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{H} &= 2f_{B^*} \lambda_{\Lambda_b} m_{B^*} m_{\Lambda_b}^2 (m_{\Lambda_b} - \mathcal{V})(m_N + m_{N^*}) \left( m_{B^*}^2 + m_{\Lambda_b}^2 - m_N m_{\Lambda_b} - m_N^2 \right. \\
& + \left. m_{N^*} (m_N + m_{\Lambda_b}) - m_{N^*}^2 \right), \\
\mathcal{V} &= (m_N - m_{N^*}). \tag{15}
\end{aligned}$$

### 3 Numerical results

To numerically analyze the sum rules for the strong coupling form factors and to find their behavior with respect to  $Q^2 = -q^2$  we need some inputs as presented in table 1. Besides, we also need to determine the working regions corresponding to the four auxiliary parameters,  $M^2$ ,  $M'^2$ ,  $s_0$  and  $s'_0$ . The  $M^2$  and  $M'^2$  emerge from the double Borel transformation and  $s_0$  and  $s'_0$  originate from continuum subtraction. These are auxiliary parameters, therefore, we

| Parameters  | Values                                       |
|---|--|
| $\langle\bar{u}u\rangle(1\text{ GeV}) = \langle\bar{d}d\rangle(1\text{ GeV})$ | $-(0.24 \pm 0.01)^3 \text{ GeV}^3$ [30]      |
| $\langle\frac{\alpha_s G^2}{\pi}\rangle$                                      | $(0.012 \pm 0.004) \text{ GeV}^4$ [31]       |
| $m_0^2(1\text{ GeV})$   | $(0.8 \pm 0.2) \text{ GeV}^2$ [31]           |
| $m_b$   | $(4.18 \pm 0.03) \text{ GeV}$ [32]           |
| $m_c$   | $(1.275 \pm 0.025) \text{ GeV}$ [32]         |
| $m_d$   | $4.8_{-0.3}^{+0.5} \text{ MeV}$ [32]         |
| $m_u$   | $2.3_{-0.5}^{+0.7} \text{ MeV}$ [32]         |
| $m_{B^*}$   | $(5325.2 \pm 0.4) \text{ MeV}$ [32]          |
| $m_{D^*}$   | $(2006.96 \pm 0.10) \text{ MeV}$ [32]        |
| $m_N$   | $(938.272046 \pm 0.000021) \text{ MeV}$ [32] |
| $m_{N^*}$   | $1525 \text{ TO } 1535 \text{ MeV}$ [32]     |
| $m_{\Lambda_b}$   | $(5619.5 \pm 0.4) \text{ MeV}$ [32]          |
| $m_{\Lambda_c}$   | $(2286.46 \pm 0.14) \text{ MeV}$ [32]        |
| $m_{\Sigma_b}$  | $(5811.3 \pm 1.9) \text{ MeV}$ [32]          |
| $m_{\Sigma_c}$  | $(2452.9 \pm 0.4) \text{ MeV}$ [32]          |
| $f_{B^*}$   | $(210.3_{-1.8}^{+0.1}) \text{ MeV}$ [33]     |
| $f_{D^*}$   | $(241.9_{-12.1}^{+10.1}) \text{ MeV}$ [33]   |
| $\lambda_N^2$   | $0.0011 \pm 0.0005 \text{ GeV}^6$ [34]       |
| $\lambda_{N^*}$   | $0.019 \pm 0.0006 \text{ GeV}^3$ [35]        |
| $\lambda_{\Lambda_b}$   | $(3.85 \pm 0.56)10^{-2} \text{ GeV}^3$ [36]  |
| $\lambda_{\Sigma_b}$  | $(0.062 \pm 0.018) \text{ GeV}^3$ [37]       |
| $\lambda_{\Lambda_c}$   | $(3.34 \pm 0.47)10^{-2} \text{ GeV}^3$ [36]  |
| $\lambda_{\Sigma_c}$  | $(0.045 \pm 0.015) \text{ GeV}^3$ [37]       |

Table 1: Input parameters used in the calculations.

need a region of them through which the strong coupling form factors have weak dependency on these parameters. The continuum thresholds are in relation with the first excited states in the initial and final channels. To determine them the energy that characterizes the beginning of the continuum is considered. Table 2 presents intervals of the continuum thresholds used in the calculations. To determine the working regions for the Borel mass parameters, we need to take into account the criteria that the contributions of the higher states and continuum are sufficiently suppressed and the contributions of the operators with higher dimensions are small. The intervals obtained based on these considerations are also given in table 2.

The determination of the working regions of auxiliary parameters is followed by the usage of them together with the other input parameters to obtain the variation of the coupling form factors as a function of  $Q^2$ . For this purpose, the following fit function is applied

$$g_{BN\mathcal{M}}(Q^2) = c_1 + c_2 \exp\left[-\frac{Q^2}{c_3}\right]. \quad (16)$$

where  $c_1$ ,  $c_2$  and  $c_3$  for different vertices are given in tables 3-6. This fit function is used to attain the coupling constants at  $Q^2 = -m_{\mathcal{M}}^2$  for all structures. The results for coupling



| Vertex                  | $s_0(GeV^2)$                | $s'_0(GeV^2)$              | $M^2(GeV^2)$          | $M'^2(GeV^2)$        |
|-------------------------|-----------------------------|----------------------------|-----------------------|----------------------|
| $\Lambda_b N^{(*)} B^*$ | $32.71 \leq s_0 \leq 35.04$ | $1.04 \leq s'_0 \leq 1.99$ | $10 \leq M^2 \leq 20$ | $1 \leq M'^2 \leq 3$ |
| $\Sigma_b N^{(*)} B^*$  | $34.91 \leq s_0 \leq 37.40$ | $1.04 \leq s'_0 \leq 1.99$ | $10 \leq M^2 \leq 20$ | $1 \leq M'^2 \leq 3$ |
| $\Lambda_c N^{(*)} D^*$ | $5.71 \leq s_0 \leq 6.72$   | $1.04 \leq s'_0 \leq 1.99$ | $2 \leq M^2 \leq 6$   | $1 \leq M'^2 \leq 3$ |
| $\Sigma_c N^{(*)} D^*$  | $6.51 \leq s_0 \leq 7.62$   | $1.04 \leq s'_0 \leq 1.99$ | $2 \leq M^2 \leq 6$   | $1 \leq M'^2 \leq 3$ |

Table 2: Working intervals for auxiliary parameters.

constants are presented in table 7. The presented errors in the results arise due to the uncertainties of the input parameters as well as uncertainties coming from the determination of the working regions of the auxiliary parameters. From this table we see that the maximum value belongs to the coupling constant  $g_2^*$  associated to the vertex  $\Lambda_b N^* B^*$  and the minimum value corresponds to the coupling  $g_1$  related to the  $\Lambda_c N D^*$  vertex.

|              | $c_1$             | $c_2$             | $c_3(GeV^2)$        |
|--------------|-------------------|-------------------|---------------------|
| $g_1(Q^2)$   | $-2.44 \pm 0.68$  | $-0.34 \pm 0.10$  | $-17.88 \pm 5.18$   |
| $g_2(Q^2)$   | $22.92 \pm 6.64$  | $3.87 \pm 1.12$   | $16.85 \pm 4.89$    |
| $g_1^*(Q^2)$ | $-6.21 \pm 1.73$  | $-26.76 \pm 8.01$ | $-193.72 \pm 56.17$ |
| $g_2^*(Q^2)$ | $88.27 \pm 25.60$ | $9.65 \pm 2.70$   | $24.32 \pm 7.05$    |

Table 3: Parameters appearing in the fit function of the coupling form factor related to the  $\Lambda_b N^{(*)} B^*$  vertex.

In conclusion, we calculated the strong coupling constants related to the vertices  $\Lambda_b N B^*$ ,  $\Lambda_b N^* B^*$ ,  $\Sigma_b N B^*$ ,  $\Sigma_b N^* B^*$ ,  $\Lambda_c N D^*$ ,  $\Lambda_c N^* D^*$ ,  $\Sigma_c N D^*$  and  $\Sigma_c N^* D^*$  in the framework QCD sum rules. Our results may be checked via other non-perturbative approaches. The presented results can be helpful to explain different exotic events observed via different experiments. These results may also be useful in the analysis of the results of heavy ion collision experiments as well as in exact determinations of the modifications in the masses, decay constants and other parameters of the  $B^*$  and  $D^*$  mesons in nuclear medium.

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|              | $c_1$              | $c_2$               | $c_3(\text{GeV}^2)$ |
|--------------|--------------------|---------------------|---------------------|
| $g_1(Q^2)$   | $297.08 \pm 89.12$ | $-282.66 \pm 81.97$ | $-225.11 \pm 67.53$ |
| $g_2(Q^2)$   | $-18.12 \pm 5.07$  | $-3.82 \pm 1.14$    | $14.06 \pm 4.08$    |
| $g_1^*(Q^2)$ | $87.60 \pm 25.40$  | $-82.34 \pm 23.87$  | $-24.88 \pm 7.21$   |
| $g_2^*(Q^2)$ | $31.80 \pm 9.22$   | $0.90 \pm 0.26$     | $-6.70 \pm 1.94$    |

Table 4: Parameters appearing in the fit function of the coupling form factor related to the  $\Sigma_b N^{(*)} B^*$  vertex.

|              | $c_1$            | $c_2$                     | $c_3(\text{GeV}^2)$ |
|--------------|------------------|---------------------------|---------------------|
| $g_1(Q^2)$   | $1.28 \pm 0.36$  | $0.92 \pm 0.27$           | $-155.98 \pm 45.24$ |
| $g_2(Q^2)$   | $3.88 \pm 1.13$  | $1.27 \pm 0.38$           | $3.60 \pm 1.04$     |
| $g_1^*(Q^2)$ | $3.01 \pm 0.87$  | $(17.97 \pm 5.21)10^{-4}$ | $-2.77 \pm 0.80$    |
| $g_2^*(Q^2)$ | $11.52 \pm 3.23$ | $-2.38 \pm 0.71$          | $2.26 \pm 0.66$     |

Table 5: Parameters appearing in the fit function of the coupling form factor related to the  $\Lambda_c N^{(*)} D^*$  vertex.

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|              | $c_1$            | $c_2$                    | $c_3(\text{GeV}^2)$ |
|--------------|------------------|--------------------------|---------------------|
| $g_1(Q^2)$   | $-6.94 \pm 2.01$ | $11.18 \pm 3.35$         | $8.30 \pm 2.41$     |
| $g_2(Q^2)$   | $-4.64 \pm 1.35$ | $-(1.41 \pm 0.4)10^{-2}$ | $1.53 \pm 0.45$     |
| $g_1^*(Q^2)$ | $26.37 \pm 7.65$ | $-23.18 \pm 6.49$        | $-11.03 \pm 3.20$   |
| $g_2^*(Q^2)$ | $15.47 \pm 4.48$ | $2.22 \pm 0.67$          | $-6.34 \pm 1.84$    |

Table 6: Parameters appearing in the fit function of the coupling form factor related to the  $\Sigma_c N^{(*)} D^*$  vertex.

| Vertex                  | $ g_1(Q^2 = -m_{\mathcal{M}}^2) $ | $ g_2(Q^2 = -m_{\mathcal{M}}^2) $ | $ g_1^*(Q^2 = -m_{\mathcal{M}}^2) $ | $ g_2^*(Q^2 = -m_{\mathcal{M}}^2) $ |
|-------------------------|-----------------------------------|-----------------------------------|-------------------------------------|-------------------------------------|
| $\Lambda_b N^{(*)} B^*$ | $2.51 \pm 0.75$                   | $43.73 \pm 13.11$                 | $29.32 \pm 8.78$                    | $119.26 \pm 35.77$                  |
| $\Sigma_b N^{(*)} B^*$  | $47.87 \pm 14.35$                 | $46.83 \pm 14.04$                 | $61.25 \pm 18.37$                   | $31.81 \pm 9.54$                    |
| $\Lambda_c N^{(*)} D^*$ | $2.05 \pm 0.61$                   | $7.78 \pm 2.33$                   | $3.01 \pm 0.90$                     | $2.60 \pm 0.78$                     |
| $\Sigma_c N^{(*)} D^*$  | $11.21 \pm 3.36$                  | $4.64 \pm 1.39$                   | $10.29 \pm 3.08$                    | $16.65 \pm 4.99$                    |

Table 7: Values of the strong coupling constants for the vertices under consideration.

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